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Partial Differential Equation To Estimate Blur And Noise

Arun Bhatt
Department of Electronics & Comm. Engineering,
Shri Ram Institute of Science & Technology,
Jabalpur, (M.P.)

Dr. J. P. Upadhyay
Department of Electronics & Comm. Engineering,
Shri Ram Institute of Science & Technology,
Jabalpur, (M.P.)

***Abstract*— Many fractional-order based methods have been used in image processing field, and many methods are developed to solve the problem of fractional systems. The traditional integer-order partial differential equation-based image denoising approaches often blur the edge and complex texture detail; thus, their denoising effects for texture image are not very good. To solve the problem, a fractional partial differential equation-based denoising model for texture image is proposed, which applies a novel mathematical method—fractional calculus to image processing from the view of system evolution. We know from previous studies that fractional-order calculus has some unique properties comparing to integer-order differential calculus that it can nonlinearly enhance complex texture detail during the digital image processing.**

Keywords—Topological Degree, Elliptic Problem, Homotopy, Image Restoration.

I. INTRODUCTION

Image retrieval system is a computer system for Image denoising is an important image processing task, both as a process itself, and as a component in other processes. Very many ways to de noise an image or a set of data exists. The main properties of a good image denoising model are that it will remove noise while preserving edges. Traditionally, linear models have been used. One common approach is to use a Gaussian filter, or equivalently solving the heat-equation with the noisy image as input-data, i.e. a linear, 2nd order PDE-model. For some purposes this kind of denoising is adequate. One big advantage of linear noise removal models is the speed. But a drawback of the linear models is that they are not able to preserve edges in a good manner: edges, which are recognized as discontinuities in the image, are smeared out. Nonlinear models on the other hand can handle edges in a much better way than linear models can. One popular model for nonlinear image denoising is the Total Variation (TV)-filter, introduced by Rudin, Osher and Fatemi. This filter is very good at preserving edges, but smoothly varying regions in the input image are transformed into piecewise constant regions in the output image. Using the TV-filter as a denoiser leads to solving a 2nd order non-linear PDE. Since smooth regions are transformed into piecewise constant regions when using the TV-filter, it is desirable to create a model for which smoothly varying regions are transformed into smoothly varying regions, and yet the edges are preserved. This can be done for instance by solving a 4th order PDE instead of the 2nd order PDE from the TV-filter. Results show that the 4th order filter produces much better results in smooth regions, and still preserves edges in a very good way. Some results showing the behavior of the 4th order model is shown in figure 1.

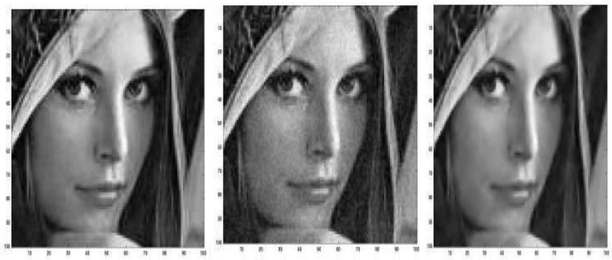

Figure 1: Fourth Order PDE Result

Here, the leftmost image is the original image, the middle image is imposed with noise, and the rightmost image is the restored image using the 4th order model. Another approach is to combine a 2nd and 4th order method. The idea here is that smooth regions are filtered by the 4th order scheme, while edges are filtered by a 2nd order scheme. To choose in which areas of the image each of the models are to be used, one has to construct a weight function. Another way of denoising images is the following: Instead of working directly with the images, the noisy normal vectors of the image are processed instead. Then, the smoothed normal vectors are used to reconstruct a denoised image. This approach gives very good results. The process is illustrated in figure 2, figure 3 and Figure 4:

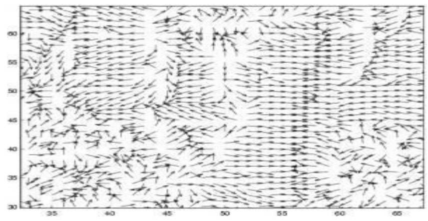

Figure 2: Result 1

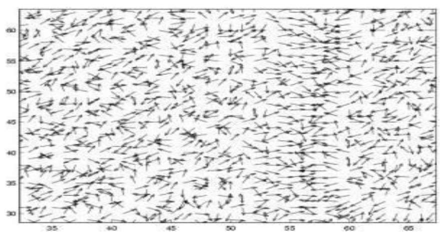

Figure 3: Result 2

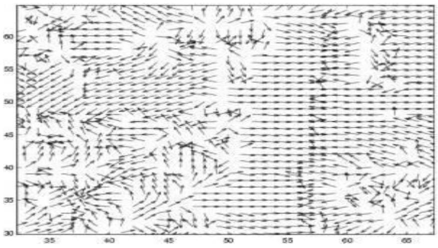

Figure 4: Result 3

The three images above show a small excerpt of the normal vectors of the above shown image. The first image shows the normal of the original image, the middle image shows the normal of the noisy image, and the last image shows the smoothed normal.

II. IMAGE DENOISING METHODS

Denoising of image data has been an active area of research, with several different approaches being proposed using techniques such as wavelets, isotropic and anisotropic diffusion, bilateral filtering, etc. We observe that image contains a large amount of redundancy in plain areas where adjacent picture elements have almost the same values which means the pixel values are highly correlated.

The mean filter is used in applications where the noise in certain regions of the image needs to be removed. In other words, the mean filter is useful when only a part of the image needs to be processed.

Figure 4: Input to Mean Filter corrupted with salt and pepper noise

Figure 5: Image after Mean Filtering

When the image is corrupted with salt and pepper noise, it looks as shown in Image 3.3. When Figure 5 is subjected to the LMS adaptive filtering, it gives an output image shown in Figure 6. Similar to the mean filter, the LMS adaptive filter works well for images corrupted with salt and pepper type noise. But this filter does a better denoising job compared to the mean filter.

Figure 6: Input to LMS adaptive Filter corrupted with salt and pepper noise

Figure 7: Image after LMS adaptive Filtering

The median filter also follows the moving window principle similar to the mean filter. A 3×3, 5×5, or 7×7 kernel of pixels is scanned over pixel matrix of the entire image. The median of the pixel values in the window is computed, and the center pixel of the window is replaced with the computed median. Median filtering is done by, first sorting all the pixel values from the surrounding neighborhood into numerical order and then replacing the pixel being considered with the middle pixel value. Note that the median value must be written to a separate array or buffer so that the results are not corrupted as the process is performed. Figure 3.7 illustrates the methodology.

Neighborhood values:

115,119,120,123,124,125,126,127,150

Median value: 124

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Figure 8: Concept of median filtering

Figure 8 is the image corrupted with salt and pepper noise and is given to the function medfilt2() for median filtering. The window specified is of size 3×3. Figure 10 is the output after median filtering. It can be observed that the edges are preserved and the quality of denoising is much better compared to the previous images.

Figure 9: Input to median filter

Figure 10 Output from median filter

III. PROPOSED WORK

A digital image is a collection of pixels arranged in a rectangular two dimensional (2D) array. For a gray-scale (black and white) image, we obtain a scalar intensity value at each of the pixel locations. These intensity values are usually quantized between 0 and 255. Hence, a digital image corresponds to a matrix of discrete values in the range of [0 - 255]. Consider the example of figure 11.

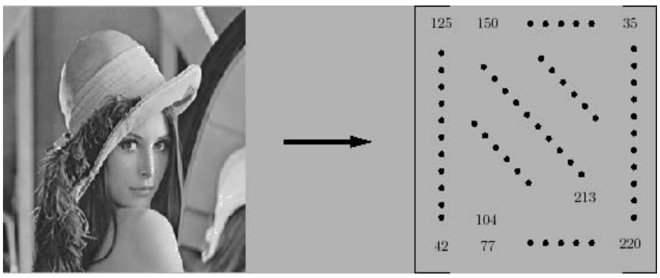

Figure 11: A digital image is essentially a matrix of integers in [0 -255]

Images captured of the real world objects (scenes) are prone to random fluctuations in the observed intensity values. This variation in the intensity values is referred to as image noise. It is an unavoidable by-product of the image capture process and it predominantly arises from the sensor and circuitry of the digital camera. Figure 12 shows a noisy image.

Figure 12: (left) ``office'' image WITHOUT noise. (Right) ``office'' image corrupted WITH noise.

The process of removing noise from an image is known as noise reduction or denoising. A standard denoising technique is the convolution of the image with a 2D Gaussian distribution. The formula is shown below:

$$G(x,Y)=1/\sqrt{2\pi\sigma^2}\ \ \exp^{-(x2 + y2)/2\sigma 2}$$

Smoothing is a process by which data points are averaged with their neighbors in a series, such as a time series, or image. This (usually) has the effect of blurring the sharp edges in the smoothed data. Smoothing is sometimes referred to as filtering, because smoothing has the effect of suppressing high frequency signal and enhancing low frequency signal.

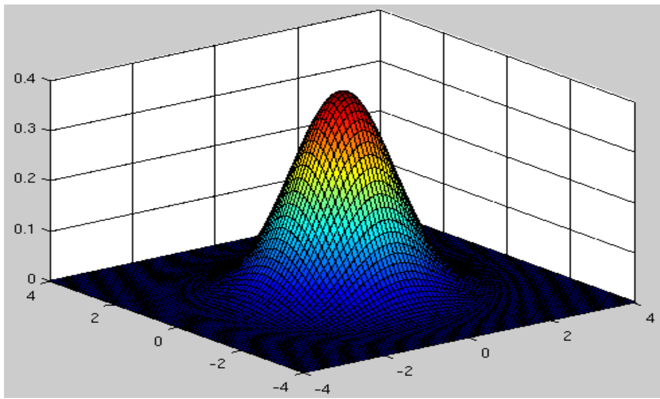

Figure 13: A zero mean 2D Gaussian with σ=1

In practice as the image contains discrete pixel locations, the Gaussian distribution needs to be approximated using a convolution kernel before the convolution operation can be

0.003	0.0133	0.0219	0.0133	0.003
0.0133	0.0596	0.0983	0.0596	0.0133
0.0219	0.0983	0.1621	0.0983	0.0219
0.0133	0.0596	0.0983	0.0596	0.0133
0.003	0.0133	0.0219	0.0133	0.003

performed. Figure 4.4 shows a 2D Gaussian convolution kernel.

Figure 14: The 5*5 convolution kernel of a Gaussian with $\sigma=1$

we consider a linear isotropic diffusion process on an image domain for the task of denoising. The linear isotropic diffusion process can be described by:

$$\partial u/\partial t=\mathrm{div}(d\Delta u)$$

$$u(x,y,0)=I(x,y)$$

Where d is a scalar constant diffusivity, I(x,y) is the initial noisy image, u(x,y,t) is the image obtained after a diffusion time t . Note that here u(x, y,t) represents the evolving intensity distribution corresponding to the evolving concentration distribution c(x,y,t) .

IV. RESULT AND IMPLEMENTATION

In this section we explain our result. For this we taken five parameters Average absolute difference, signal to noise ratio (SNR), peak signal to noise ratio (PSNR), Image Fidelity and Mean square error as shown in table 1.

We start the comparison taking consideration with blur as shown in figure 15 and the result is shown in table 1.

Figure 15: Considering Blur Parameter

Table 1: Comparison Considering Blur Parameters

Parameters	Proposed (Blur)	Proposed (Noise)	Proposed (Blur + Noise)
Average absolute difference	0.000000	0.002409	0.000593
signal to noise ratio (SNR),	259	13.56	14.34
peak signal to noise ratio (PSNR)	297	49.45	50.46
Image Fidelity	0.000253	-0.041	-0.089
Mean square error	0.000000	0.002677	0.004275

Then we consider images with noise parameters as shown in figure 16 and the result is shown in table 2. If we compare the result of table 3 with table 2, it shows good result.

Figure 16:Considering Noise Parameter

Table 2: Comparison Considering Noise Parameters

Parameters	Proposed (Blur)	Proposed (Noise)	Proposed (Blur + Noise)
Average absolute difference	0.006374	0.0033	Null
Signal to noise ratio (SNR),	5.64	14.45	Null
Peak signal to noise ratio (PSNR)	42.89	50.78	Null
Image Fidelity	-0.31	-0.039	Null
Mean square error	0.0154	0.0022	

Then we consider images with noise and Blur parameters as shown in figure 17 and the result is shown in table3.

Figure 17:Considering Noise and Blur Parameters

Table 3: Considering Blur and Noise (Salt and Pepper) Parameters

Parameters	Proposed (Blur)	Proposed (Noise)	Proposed (Blur + Noise)
Average absolute difference	0.001313	0.002445	0.000638
signal to noise ratio (SNR),	-44.00	11.83	12.87
peak signal to noise ratio (PSNR)	-6.80	48.14	49.17
Image Fidelity	-18998	-0.06	-0.05
Mean square error	1193	0.0039	0.0030

After performing the above comparison we can show that the result is better in comparison to the traditional methods.

V. CONCLUSION AND FUTURE WORK

We observe that image contains a large amount of redundancy in plain areas where adjacent picture element has almost the same values which means the pixel values are highly correlated. In addition, image can contain subjective redundancy, which is determined by properties of a human visual system (HVS).

In this dissertation we have presented a method for image denoising. The process of removing noise from an image is known as noise reduction or denoising. A standard denoising technique is the convolutions of the image with a 2D Gaussian distribution. We apply sampling and convolution which is based on Weiner filters. We also provide comparison on the basis of five parameters Average absolute difference, signal to noise ratio (SNR), peak signal to noise ratio (PSNR), Image Fidelity and Mean square error. The result is better in comparison to the previous technique.In this dissertation we have presented a method for image denoising which is based on noise and blur both. A standard denoising technique is the convolutions of the image with a 2D Gaussian distribution. We apply sampling and convolution which is based on Weiner filters and PDE.

Since selection of the right denoising procedure plays a major role, it is important to experiment and compare the methods. As future research, we would like to work further on the comparison of the denoising techniques. If the features of the denoised signal are fed into a neural network pattern recognizer, then the rate of successful classification should determine the ultimate measure by which to compare various denoising procedures. Besides, the complexity of the algorithms can be measured according to the CPU computing time flops. This can produce a time complexity standard for each algorithm. These two points would be considered as an extension to the present work done

REFERENCES

[1] D.L. Donoho, De-Noising by Soft Thresholding, IEEE Trans. Info. Theory 43, pp. 933-936, 1993.

[2] Javier Portilla, Vasily Strela, Martin J. Wainwright, Eero P. Simoncelli, Adaptive Wiener Denoising using a Gaussian Scale Mixture Model in the wavelet Domain, Proceedings of the 8th International Conference of Image Processing Thessaloniki, Greece. October 2019.

[3] S. Grace Chang, Bin Yu and M. Vattereli,Adaptive Wavelet Thresholding for Image Denoising and Compression, IEEE Trans. Image Processing, vol. 9, pp. 1532-1546, Sept. 2019.

[4] D.L. Donoho and I.M. Johnstone, Adapting to unknown smoothness via wavelet shrinkage, Journal of American Statistical Assoc., Vol. 90, no. 432, pp 1200-1224, Dec. 2018.

[5] S. Grace Chang, Bin Yu and M. Vattereli, Wavelet Thresholding for Multiple Noisy Image Copies, IEEE Trans. Image Processing, vol. 9, pp.1631- 1635, Sept.2018.

[6] S. Grace Chang, Bin Yu and M. Vattereli, Spatially Adaptive Wavelet Thresholding with Context Modeling for Image Denoising,, IEEE Trans. Image Processing, vol. 9, pp. 1522-1530, Sept. 2019.

[7] M. Vattereli and J. Kovacevic, Wavelets and Subband Coding. Englewood Cliffs, NJ,Prentice Hall, 2015.

[8] Tongzhou Zhao, Yanli Wang, Ying Ren, Yalan Liao, "Approach of Image Denoising Based on Discrete Multi-wavelet Transform", vol. 9, pp.163-176, Sept 2012, IEEE.

[9] Sudipta Roy et.al : " An Efficient Denoising Model based on Wavelet and Bilateral Filters",International Journal of Computer Applications (0975 – 8887) Volume 53– No.10, September 2012 page no. 28-35.

[10] P.Saravanan et.al.: "An Optimal Noise Removal Approach for Lateral Skull Images", IJITR,Volume No. 1, Issue No. 2, February - March 2013, 182 – 186.

[11]. J UMAMAHESWARI et.al.: " Hybrid Denoising Method for Removal of Mixed Noise in Medical Images",(IJACSA) International Journal of Advanced Computer Science and Applications, Vol. 3, No. 5, 2012 page no. 44-47

[12]Chandrika Saxena,et.al.:"Noises and Image Denoising Techniques: A Brief Survey" International Journal of Emerging Technology and Advanced Engineering Volume 4, Issue 3, March 2014 page no.878-885.

[13]. Michael Elad et.al.: "Image Denoising Via Sparse and Redundant Representations Over Learned Dictionaries" IEEE Transactions On Image Processing, Vol. 15, No. 12, December 2006 page no. 3736-3745.

[14]. Florian Luisier et.al.:"Image Denoising in Mixed Poisson–Gaussian Noise" IEEE Transactions On Image Processing, VOL. 20, NO. 3, MARCH 2011 page no.696-708.

[15].Gabriela Ghimpe et.al : "A Decomposition Framework for Image Denoising Algorithms" IEEE Transactions On Image Processing, 2016 Vol. 25, No. 1.

[16]Abdolhossein Fathi and Ahmad Reza Naghsh-Nilchi, "Efficient Image Denoising Method Based on a New Adaptive Wavelet Packet Thresholding Function", IEEE Transactions on Image Processing, Vol. 21, No. 9, September 2019.

[17]. Yongqin Zhang et.al: "Joint image denoising using adaptive principal component analysis and self-similarity" Information Sciences 259 (2014) 128–141.

[18]. R.Sudhakar et.al : "Image Compression using Coding of Wavelet Coefficients – A Survey" ICGST-GVIP Journal, Volume (5), Issue (6), June 2005.

[19] Sonja Grgic, Mislav Grgic, 'Performance Analysis of Image Compression Using Wavelets', IEEE International Symposium on Industrial Electronics, Bled, Slovenia, Vol 43, No 3, June 2001.

[20] P.-L. Shui, Z.-F. Zhou, J.-X. Li. "Image denoising algorithm via best wavelet packet base using Wiener cost function". IET Image Process,Vol. l, No.3 Sep. 2017.

[21] Zao-Chao Bao, Xin-Ge You, Chun-Fang Xing, Qing-Yan He. "Image Deniosing by Using Non-tensor Product Wavelets Filter Banks".Proceedings of the Sixth International Conference on Machine Learning and Cybernetics, Hong Kong, 19-22 August 2017.

[22] Jiying Wu, Qiuqi Ruan. "An Adaptive Edge Enhancing Image Denoising Model Based on Fuzzy Theory". Proc. of 2007 International Symposium on Intelligent Signal Processing and Communication Systems Nov.28-Dec.1, 2007.

[23] Han Liu, Yong Guo, Gang Zheng. "Image Denoising Based on Least Squares Support Vector Machines ". Proc. of the 6th World Congress on Intelligent Control and Automation, June 21 - 23, 2016.